Areas of mathematics

Mathematics encompasses a growing variety and depth of subjects over its history, and comprehension of it requires a system to categorize and organize these various subjects into a more general **areas of mathematics**. A number of different classification schemes have arisen, and though they share some similarities, there are differences due in part to the different purposes they serve.

A traditional division of mathematics is into pure mathematics; mathematics studied for its intrinsic interest, and applied mathematics; the mathematics that can be directly applied to real-world problems.[note 1] This division is not always clear and many subjects have been developed as pure mathematics to find unexpected applications later on. Broad divisions, such as discrete mathematics, computational mathematics and so on have emerged more recently.

An ideal system of classification permits adding new areas into the organization of previous knowledge, and fitting surprising discoveries and unexpected interactions into the outline. For example, the Langlands program has found unexpected connections between areas previously thought unconnected, at least Galois groups, Riemann surfaces and number theory.

Classification systems

- Wikipedia uses a Category: Mathematics system on its articles, and also has a list of mathematics lists.

- The Mathematics Subject Classification (MSC) is produced by the staff of the review databases Mathematical Reviews and Zentralblatt MATH. Many mathematics journals ask authors to label their papers with MSC subject codes. The MSC divides mathematics into over 60 areas, with further subdivisions within each area.
- In the Library of Congress Classification, mathematics is assigned the many subclass QA within the class Q (Science). The LCC defines broad divisions, and individual subjects are assigned specific numerical values.
- The Dewey Decimal Classification assigns mathematics to division 510, with subdivisions for Algebra & Number theory, Arithmetic, Topology, Analysis, Geometry, Numerical analysis, and Probabilities & Applied mathematics.
- The Categories within Mathematics (https://arxiv.org/archive/math) list is used by the arXiv for categorizing preprints. It differs from MSC; for example, it includes things like Quantum algebra.
- The IMU uses its programme structure (http://www.mathunion.org/activities/icm/icm-2010-program-structure/) for organizing the lectures at its ICM every four years. One top-level section that MSC doesn't have is Lie theory.
- The ACM Computing Classification System includes a couple of mathematical categories (http://portal.acm.org/ccs.cfm?part=author&coll=portal&dl=GUIDE) F. Theory of Computation and G. Mathematics of Computing.
- MathOverflow has a tag system (https://mathoverflow.net/tags) .
- Mathematics book publishers such as Springer (subdisciplines (https://www.springer.com/mathematics?SGWID=0-10042-0-0-0)), Cambridge (Browse Mathematics and statistics (http://www.cambridge.org/gb/knowledge/other_subject/item1521/?site_locale=en_GB)) and the AMS (subject area (https://www.ams.org/bookstore/textbooks)) use their own subject lists on their websites to enable customers to browse books or filter searches by subdiscipline, including topics such as mathematical biology and mathematical finance as top-level headings.
- Schools and other educational bodies have syllabuses.
- SIAM divides the areas of applied mathematics in activity groups (http://siam.org/activity/) .

Major divisions of mathematics

Pure mathematics

Foundations

Mathematicians have always worked with logic and symbols, but for centuries the underlying laws of logic were taken for granted, and never expressed symbolically. **Mathematical logic**, also known as **symbolic logic**, was developed when people finally realized that the tools of mathematics can be used to study the structure of logic itself. Areas of research in this field have expanded rapidly, and are usually subdivided into several distinct subfields.

- Proof theory and constructive mathematics : Proof theory grew out of David Hilbert's ambitious program to formalize all the proofs in mathematics. The most famous result in the field is encapsulated in Gödel's incompleteness theorems. A closely related and now quite popular concept is the idea of Turing machines. **Constructivism** is the outgrowth of Brouwer's unorthodox view of the nature of logic itself; constructively speaking, mathematicians cannot assert "Either a circle is round, or it is not" until they have actually exhibited a circle and measured its roundness.
- Model theory : Model theory studies mathematical structures in a general framework. Its main tool is first-order logic.
- Set theory : A set can be thought of as a collection of distinct things united by some common feature. Set theory is subdivided into three main areas. Naive set theory is the original set theory developed by mathematicians at the end of the 19th century. Axiomatic set theory is a rigorous axiomatic theory developed in response to the discovery of serious flaws (such as Russell's paradox) in naive set theory. It treats sets as "whatever satisfies the axioms", and the notion of collections of things serves only as motivation for the axioms. Internal set theory is an axiomatic extension of set theory that supports a logically consistent identification of *illimited* (enormously large) and *infinitesimal* (unimaginably small) elements within the real numbers. See also List of set theory topics.

History and biography

The history of mathematics is inextricably intertwined with the subject itself. This is perfectly natural: mathematics has an internal organic structure, deriving new theorems from those that have come before. As each new generation of mathematicians builds upon the achievements of their ancestors, the subject itself expands and grows new layers, like an onion.

Recreational mathematics

From magic squares to the Mandelbrot set, numbers have been a source of amusement and delight for millions of people throughout the ages. Many important branches of "serious" mathematics have their roots in what was once a mere puzzle and/or game.

Number theory

Number theory is the study of numbers and the properties of operations between them. Number theory is traditionally concerned with the properties of integers, but more recently, it has come to be concerned with wider classes of problems that have arisen naturally from the study of integers.

- Arithmetic : An elementary part of number theory that primarily focuses upon the study of natural numbers, integers, fractions, and decimals, as well as the properties of the traditional operations on them: addition, subtraction, multiplication and division. Up until the 19th century, *arithmetic* and *number theory* were synonyms, but the evolution and growth of the field has resulted in arithmetic referring only to the elementary branch of number theory.
- Elementary number theory: The study of integers at a higher level than arithmetic, where the term 'elementary' here refers to the fact that no techniques from other mathematical fields are used.
- Analytic number theory : Calculus and complex analysis are used as tools to study the integers.
- Algebraic number theory : The techniques of abstract algebra are used to study the integers, as well as algebraic numbers, the roots of polynomials with integer coefficients.
- Other number theory subfields : Geometric number theory; combinatorial number theory; transcendental number theory; and computational number theory. See also the list of number theory topics.

Algebra

The study of structure begins with numbers, first the familiar natural numbers and integers and their arithmetical operations, which are recorded in elementary algebra. The deeper properties of these numbers are studied in number theory. The investigation of methods to solve equations leads to the field of abstract algebra, which, among other things, studies rings and fields, structures that generalize the properties possessed by everyday numbers. Long standing questions about compass and straightedge constructions were finally settled by Galois theory. The physically important concept of vectors, generalized to vector spaces, is studied in linear algebra. Themes common to all kinds of algebraic structures are studied in universal algebra.

- Order theory : For any two distinct real numbers, one must be greater than the other. Order theory extends this idea to sets in general. It includes notions like lattices and ordered algebraic structures. See also the order theory glossary and the list of order topics.
- General algebraic systems : Given a set, different ways of combining or relating members of that set can be defined. If these obey certain rules, then a particular algebraic structure is formed. Universal algebra is the more formal study of these structures and systems.

- Field theory and polynomials : Field theory studies the properties of fields. A field is a mathematical entity for which addition, subtraction, multiplication and division are well-defined. A polynomial is an expression in which constants and variables are combined using only addition, subtraction, and multiplication.
- Commutative rings and algebras : In ring theory, a branch of abstract algebra, a commutative ring is a ring in which the multiplication operation obeys the commutative law. This means that if *a* and *b* are any elements of the ring, then $a \times b = b \times a$. Commutative algebra is the field of study of commutative rings and their ideals, modules and algebras. It is foundational both for algebraic geometry and for algebraic number theory. The most prominent examples of commutative rings are rings of polynomials.

Combinatorics

Combinatorics is the study of finite or discrete collections of objects that satisfy specified criteria. In particular, it is concerned with "counting" the objects in those collections (enumerative combinatorics) and with deciding whether certain "optimal" objects exist (extremal combinatorics). It includes graph theory, used to describe interconnected objects (a graph in this sense is a network, or collection of connected points). See also the list of combinatorics topics, list of graph theory topics and glossary of graph theory. A *combinatorial flavour* is present in many parts of problem solving.

Geometry

Geometry deals with spatial relationships, using fundamental qualities or axioms. Such axioms can be used in conjunction with mathematical definitions for points, straight lines, curves, surfaces, and solids to draw logical conclusions. See also List of geometry topics.

- Convex geometry: Includes the study of objects such as polytopes and polyhedra. See also List of convexity topics.
- Discrete geometry and combinatorial geometry: The study of geometrical objects and properties that are discrete or combinatorial, either by their nature or by their representation. It includes the study of shapes such as the Platonic solids and the notion of tessellation.
- Differential geometry: The study of geometry using calculus. It is very closely related to differential topology. Covers such areas as Riemannian geometry, curvature and differential geometry of curves. See also the glossary of differential geometry and topology.
- Algebraic geometry: Given a polynomial of two real variables, the points on a plane where that function is zero will form a curve. An algebraic curve extends this notion to polynomials over a field in a given number of variables. Algebraic geometry may be viewed as the study of these curves. See also the list of algebraic geometry topics and list of algebraic surfaces.

    - Real algebraic geometry: The study of semialgebraic sets, i.e. real number solutions to algebraic inequalities with real number coefficients, and mappings between them.
- Arithmetic geometry: The study of schemes of finite type over the spectrum of the ring of integers. Alternatively defined as the application of the techniques of algebraic geometry to problems in number theory.
- Diophantine geometry: The study of the points of algebraic varieties with coordinates in fields that are not algebraically closed and occur in algebraic number theory, such as the field of rational numbers, number fields, finite fields, function fields, and *p*-adic fields, but not including the real numbers.

Topology

Deals with the properties of a figure that do not change when the figure is continuously deformed. The main areas are point set topology (or general topology), algebraic topology, and the topology of manifolds, defined below.

- General topology: Also called *point set topology*. Properties of topological spaces. Includes such notions as open and closed sets, compact spaces, continuous functions, convergence, separation axioms, metric spaces, dimension theory. See also the glossary of general topology and the list of general topology topics.
- Algebraic topology: Properties of algebraic objects associated with a topological space and how these algebraic objects capture properties of such spaces. (Some of these algebraic objects are examples of functors.) Contains areas like homology theory, cohomology theory, homotopy theory, and homological algebra. Homotopy deals with homotopy groups (including the fundamental group) as well as simplicial complexes and CW complexes (also called *cell complexes*). See also the list of algebraic topology topics.
- Differential topology: The field dealing with differentiable functions on differentiable manifolds, which can be thought of as an *n*:dimensional generalization of a surface in the usual 3-dimensional Euclidean space.

Analysis

Within mathematics, *analysis* is the branch that focuses on rates of change (derivatives), integrals, and multiple things changing relative to (or independently of) one another.

Modern analysis is a vast and rapidly expanding branch of mathematics that touches almost every other subdivision of the discipline, finding direct and indirect applications in topics as diverse as number theory, cryptography, and abstract algebra. It is also the language of science itself and is used across chemistry, biology, and physics, from astrophysics to X-ray crystallography.

Applied mathematics

Probability and statistics

- Probability theory: The mathematical theory of random phenomena. Probability theory studies random variables and events, which are mathematical abstractions of nondeterministic events or measured quantities. See also Category:probability theory, and the list of probability topics.
  - Stochastic processes: An extension of probability theory that studies collections of random variables, such as time series or spatial processes. See also List of stochastic processes topics, and Category:Stochastic processes.
- Statistics: The science of making effective use of numerical data from experiments or from populations of individuals. Statistics includes not only the collection, analysis and interpretation of such data, but also the planning of the collection of data, in terms of the design of surveys and experiments. See also the list of statistical topics.

Computational sciences

- Numerical analysis: Many problems in mathematics cannot in general be solved exactly. Numerical analysis is the study of iterative methods and algorithms for approximately solving problems to a specified error bound. Includes numerical differentiation, numerical integration and numerical methods; c.f. scientific computing. See also List of numerical analysis topics.
- Computer algebra: This area is also called **symbolic computation** or **algebraic computation**. It deals with exact computation, for example with integers of arbitrary size, polynomials or elements of finite fields. It includes also the computation with non numeric mathematical objects like polynomial ideals or series.

Mathematical physics

- Classical Mechanics: Addresses and describes the motion of macroscopic objects, from projectiles to parts of machinery, and astronomical objects, such as spacecraft, planets, stars and galaxies.
- Mechanics of structures: Mechanics of structures is a field of study within applied mechanics that investigates the behavior of structures under mechanical loads, such as bending of a beam, buckling of a column, torsion of a shaft, deflection of a thin shell, and vibration of a bridge.
- Mechanics of deformable solids: Most real-world objects are not point-like nor perfectly rigid. More importantly, objects change shape when subjected to forces. This subject has a very strong overlap with continuum mechanics, which is concerned with continuous matter. It deals with such notions as stress, strain and elasticity.

- Fluid mechanics: Fluids in this sense includes not just liquids, but flowing gases, and even solids under certain situations. (For example, dry sand can behave like a fluid). It includes such notions as viscosity, turbulent flow and laminar flow (its opposite).
- Particle mechanics: In mathematics, a particle is a point-like, perfectly rigid, solid object. Particle mechanics deals with the results of subjecting particles to forces. It includes celestial mechanics—the study of the motion of celestial objects.

Other applied mathematics

- Operations research (OR): Also known as operational research, OR provides optimal or near-optimal solutions to complex problems. OR uses mathematical modeling, statistical analysis, and mathematical optimization.
- Mathematical programming: Mathematical programming (or mathematical optimization) minimizes (or maximizes) a real-valued function over a domain that is often specified by constraints on the variables. Mathematical programming studies these problems and develops iterative methods and algorithms for their solution.

See also

- Mathematics Subject Classification
- Glossary of areas of mathematics
- Outline of mathematics

Notes

1. *For example, the Encyclopædia Britannica Eleventh Edition groups its mathematics articles as Pure, Applied, and Biographies.*

External links

- The Divisions of Mathematics (https://web.archive.org/web/20100815034900/http://www.math.niu.edu/~rusin/known-math/index/tour_div.html) [from the Web Archive; Last modified 2006/01/25]